Toward improved mapping of Sea Surface Height

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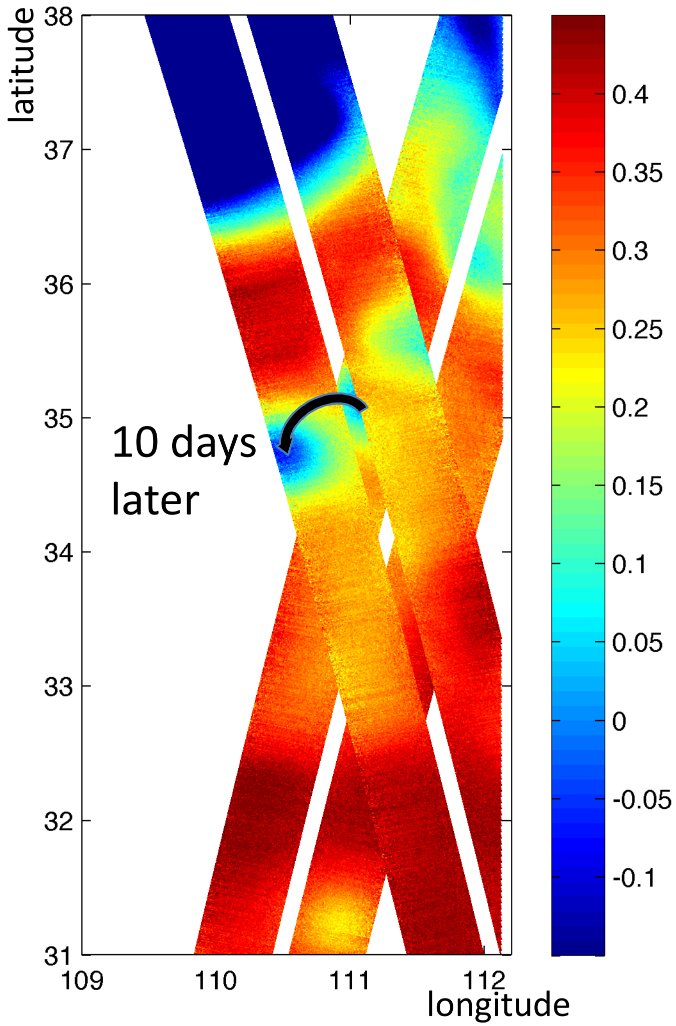
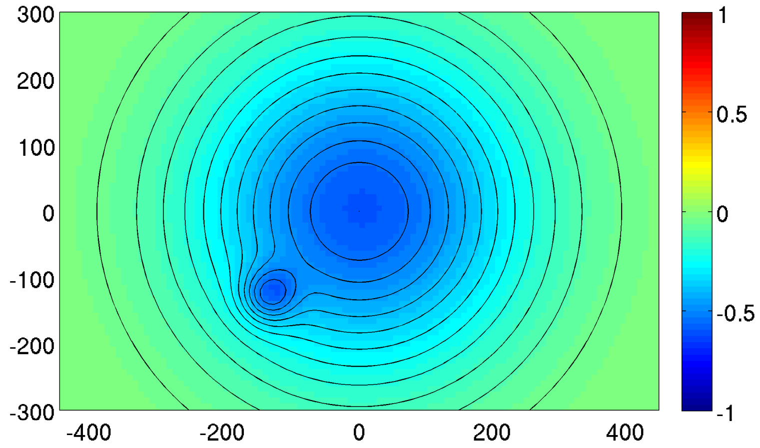
Motivations for going beyond linear objective mapping

SSH sampled with SWOT
(from SWOT-L3 simulator)



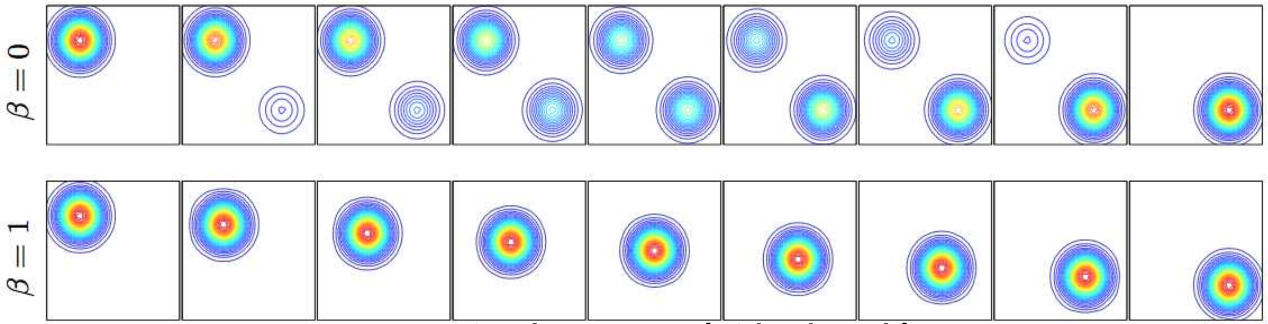
Strong non-linearities: active advection of PV



Today's OI mapping cannot handle non-linearities :

→Most of the SWOT signal would be filtered out of the maps, unused.

→ **We need to go beyond linear OI to design new data products**



Optimal transport (Vidard et al.)

First baroclinic mode PV conservation as a « propagator »

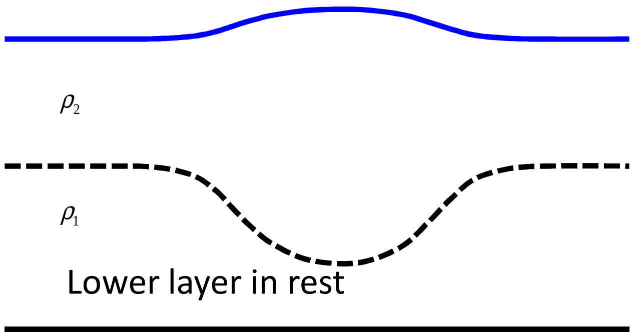
→ 1st baroclinic mode known to explain a large part of the eddy variability
→ SSH and a Rossby radius climatology are sufficient to resolve PV conservation in first baroclinic mode framework :

$$\psi = \frac{g}{f} SSH$$

$$q = \nabla^2 \psi - \frac{1}{L_R^2} \psi$$

$$\frac{\partial q}{\partial t} + J(\psi, q) - \beta \frac{\partial \psi}{\partial x} = 0$$

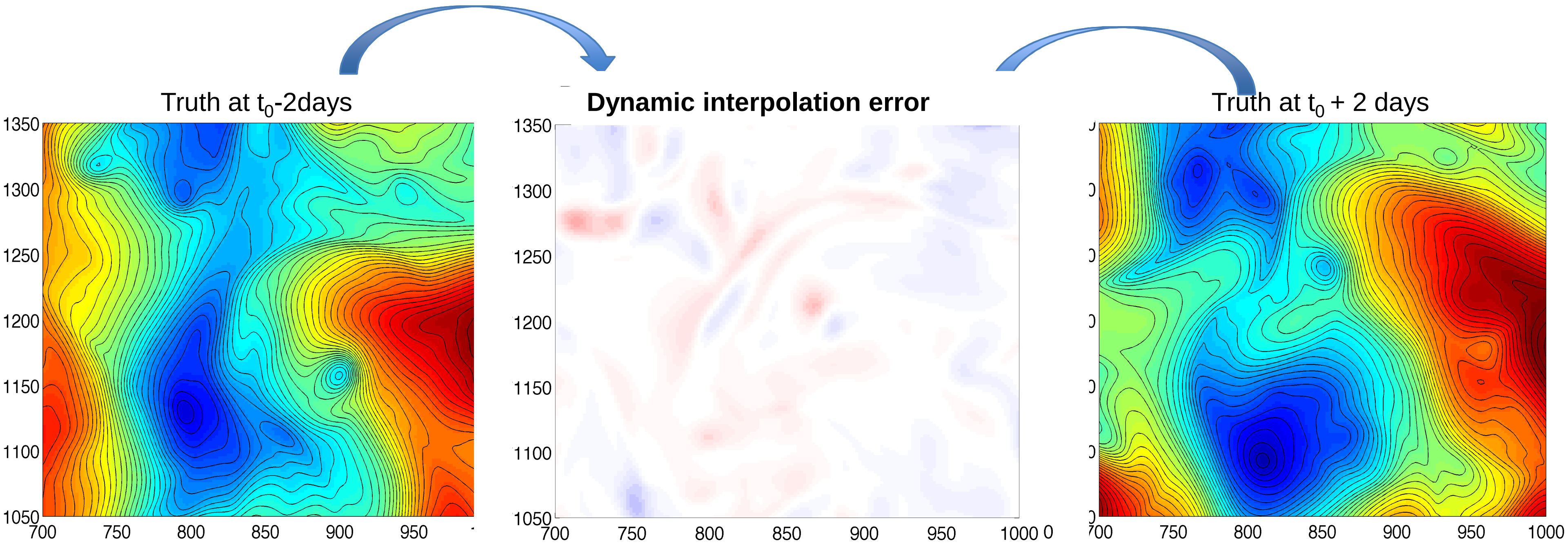
(e.g. Charney, 1948)



- **From given SSH, this system defines a short-term SSH evolution forward and backward in time**
- Physical characteristics: **- Scales >>Lr: wave-like propagation (westward)**
  - Smaller scales: self-advection

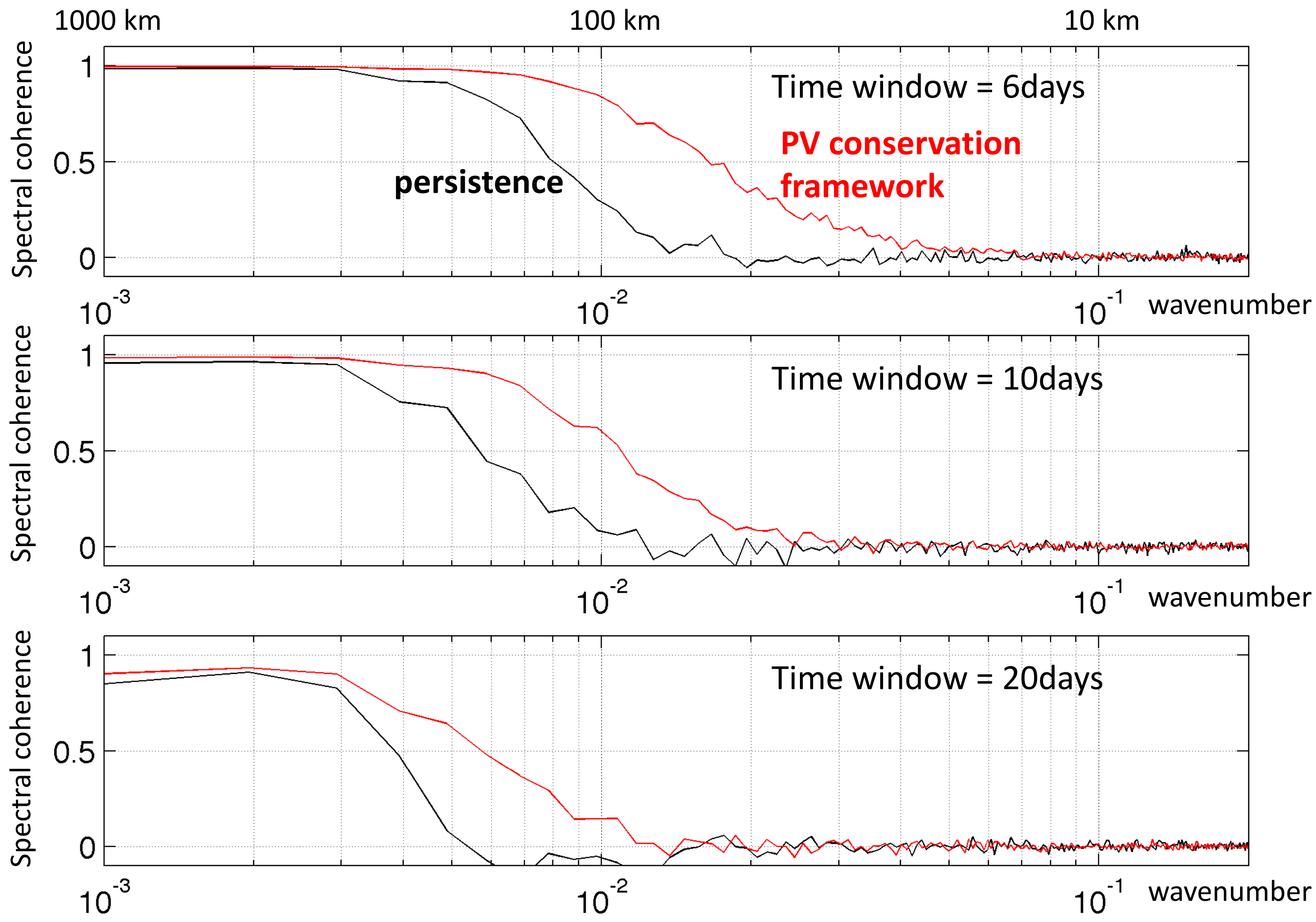
→ **We found that at short time scales (until ~10-20 days), the motion of 50km-300km mesoscale eddies is very well explained by this propagator (illustration next slide)**

Illustration on a simple case

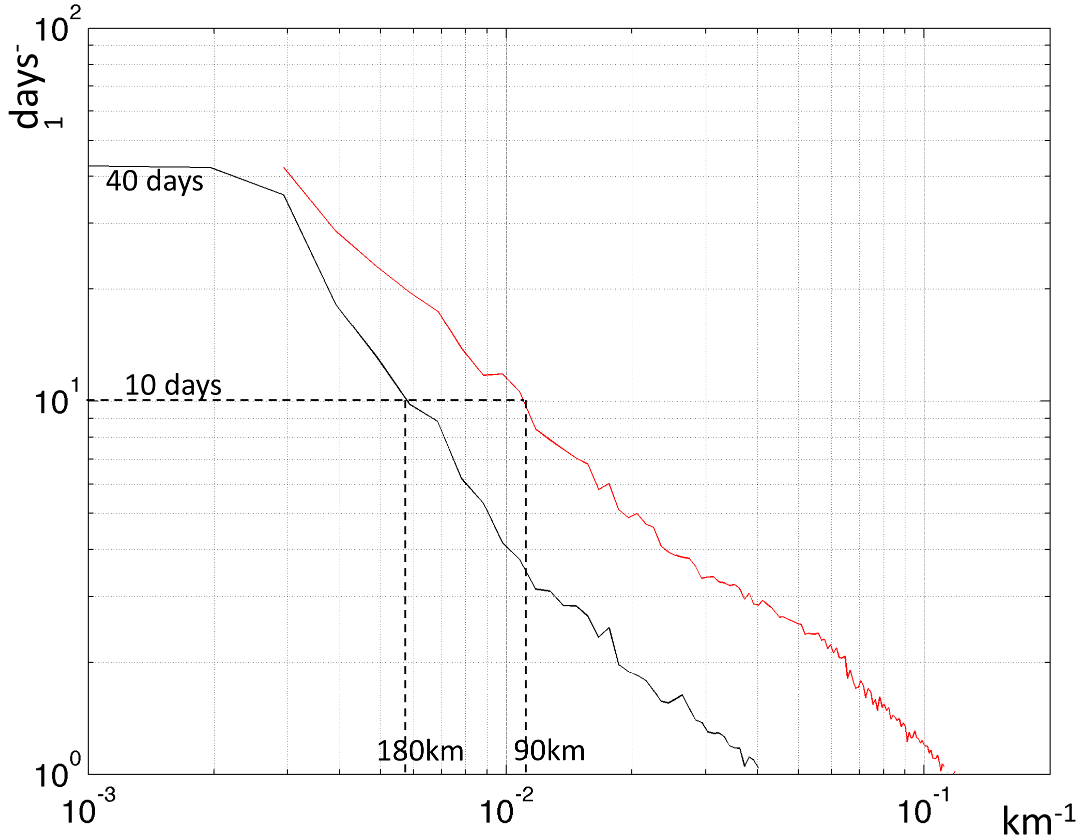


More details in Ubelmann&Klein&Fu, JTECH, 2015

- The use of the propagator significantly reduces residual errors



1000 km
100 km
10 km
Spectral coherence
1
0.5
0
Time window = 6days
PV conservation framework
persistence
10^{-3}
10^{-2}
10^{-1}
wavenumber
Time window = 10days
Time window = 20days



→ Performing interpolation in the 'propagator space' would 'extend' time decorrelations

Dynamic Optimal Interpoaltion

Standard mapping with predefined covariances B

$$\mathbf{x_a} = \mathbf{x_b} + \mathbf{BH^T}(\mathbf{HBH^T} + \mathbf{R})^{-1}[\mathbf{y_o} - \mathbf{Hx_b}]$$

Covariance model:

$$B(r,t) = \langle SSH^2 \rangle \left(1 + r + \frac{1}{6}r^2 - \frac{1}{6}r^3\right) * e^{-r} e^{-\left(\frac{t}{t_0}\right)^2}$$

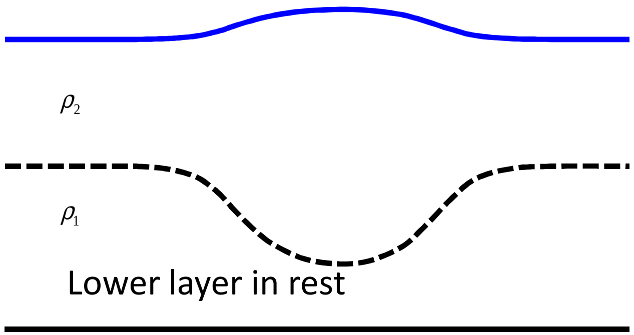
e.g. Arhan et Colin de Verdière, 1985

$$\psi = \frac{g}{f} SSH$$

$$q = \nabla^2 \psi - \frac{1}{L_R^2} \psi$$

$$\frac{\partial q}{\partial t} + J(\psi, q) - \beta \frac{\partial \psi}{\partial x} = 0$$

(e.g. Charney, 1948)



Idea: keep a centered OI time/space analysis, with above equations M used as a covariance propagator in time:

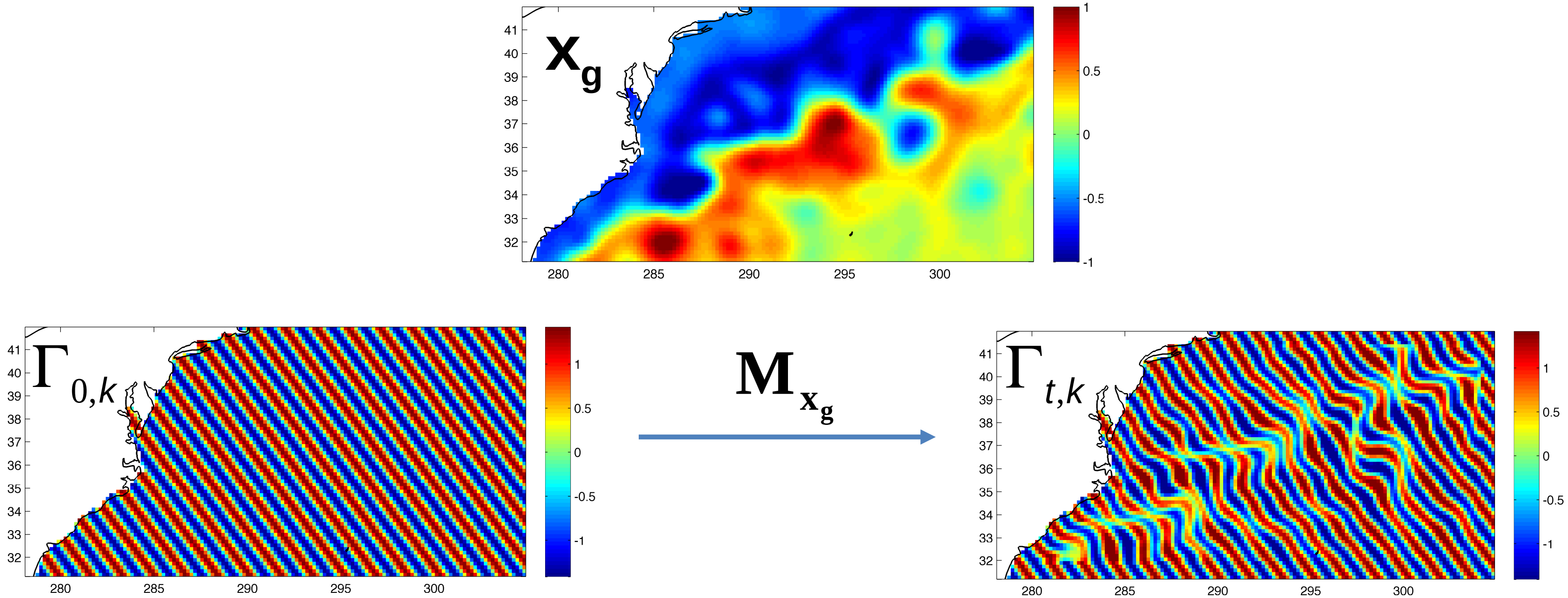
Dynamic mapping with flow-dependant covariances

$$\mathbf{x_a} = \mathbf{x_b} + \mathbf{BM_{x_g}^T H^t}(\mathbf{HM_{x_g} BM_{x_g}^T H^T} + \mathbf{R})^{-1}[\mathbf{y_o} - \mathbf{Hx_b} - \mathbf{H}(M(\mathbf{x_g} - \mathbf{x_b}) - \mathbf{M_{x_g}}(\mathbf{x_g} - \mathbf{x_b}))]$$

Practically solved in a reduced space (2D Fourier decomposition)~. Iterations on the guess xg.

Computing the covariances B’ from the linearized propagator

Time evolving guess (integrated forward and backward)



One of the ~1000 Fourier component

Linear response around the guess after 2 days

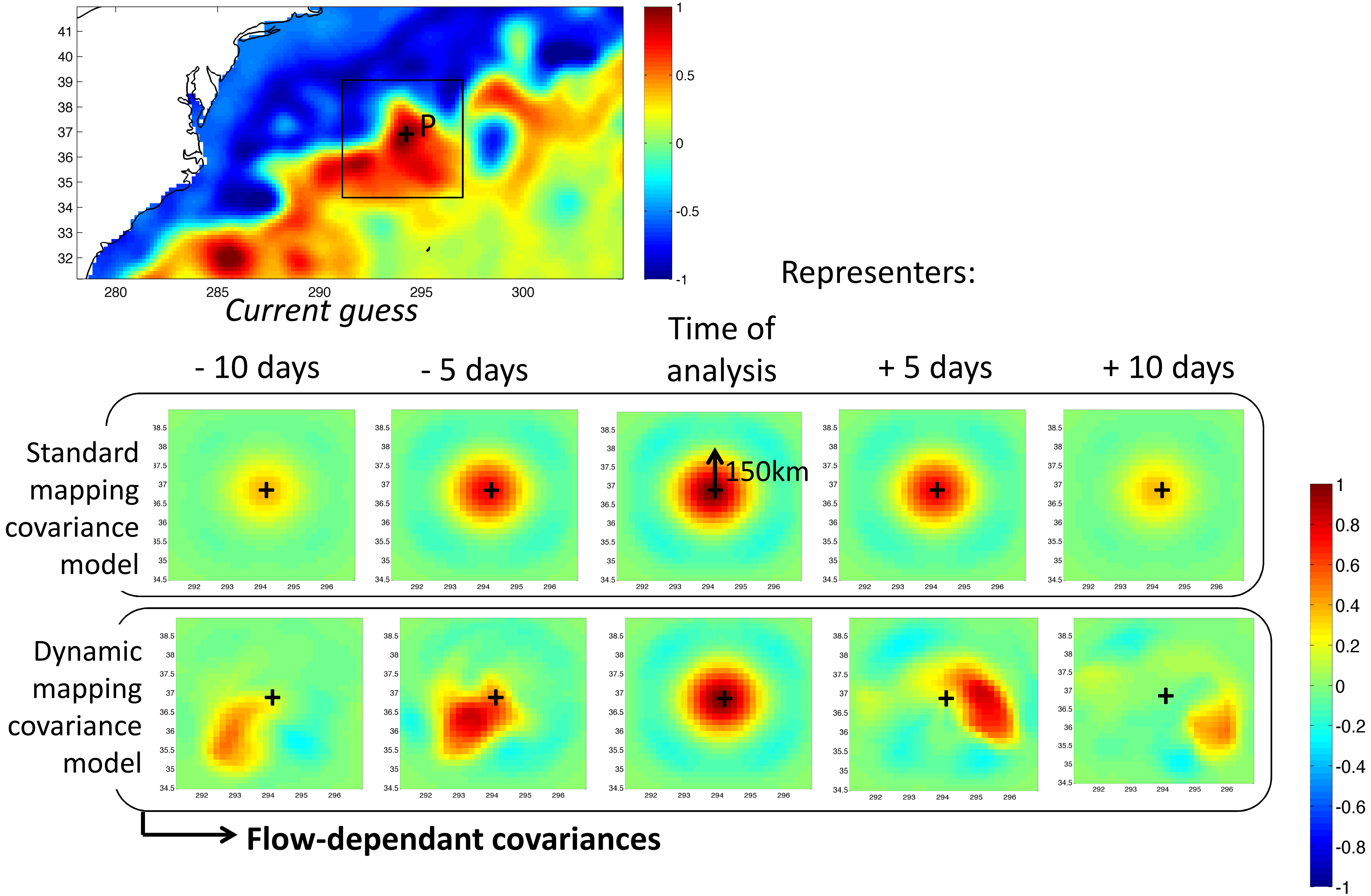
$$G_k = H\Gamma_k$$

Γ_k represents the propagated Fourier components (or ‘Green functions’) by the linear response of the propagator around the guess. G (projected in obs space) is the green function matrix

$$B' = \Gamma Q \Gamma^T$$

Q is diagonal, constructed consistently with the spatial covariance (inverse FFT of SSH spectrum)

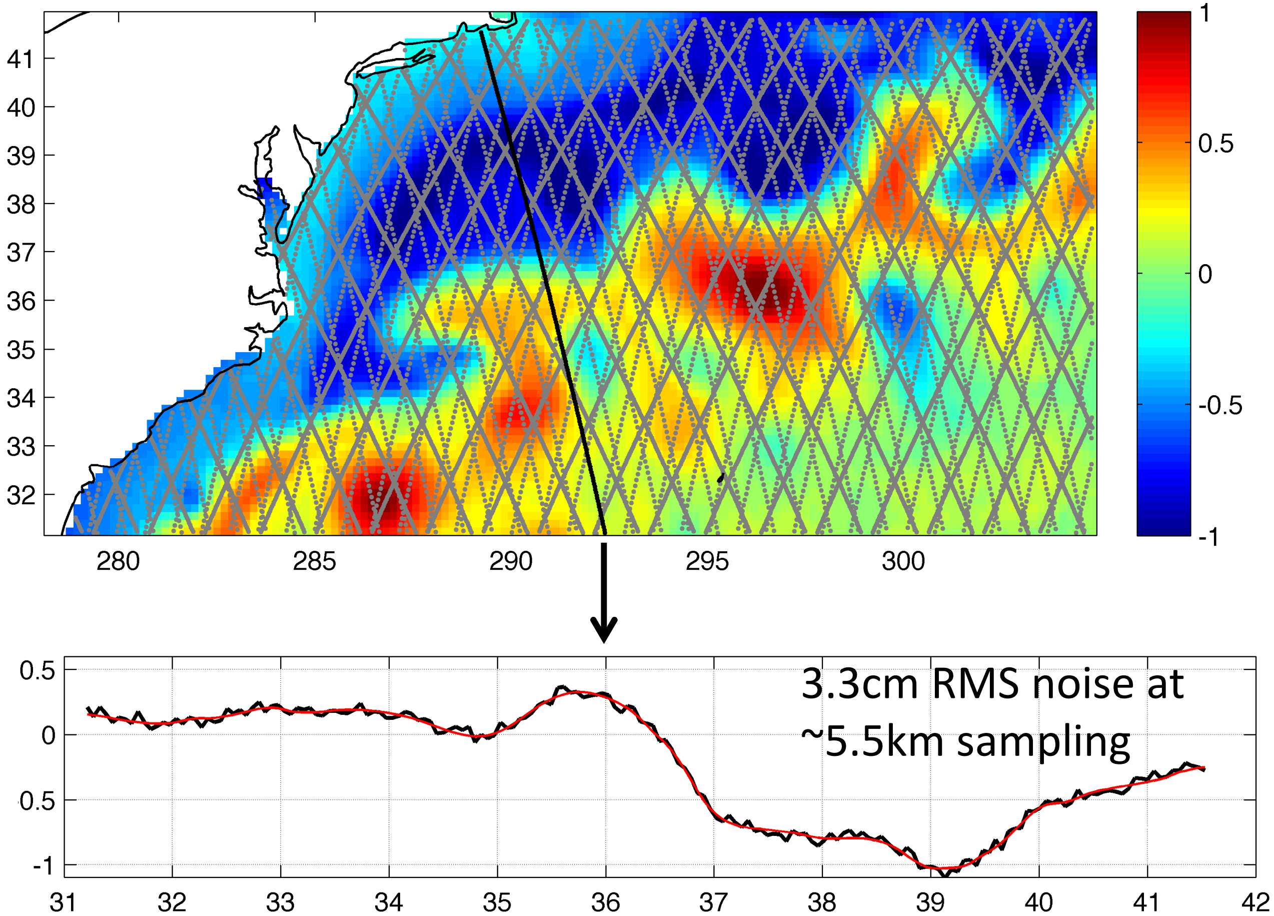
How do covariances look like



Experimental setup: OSSEs

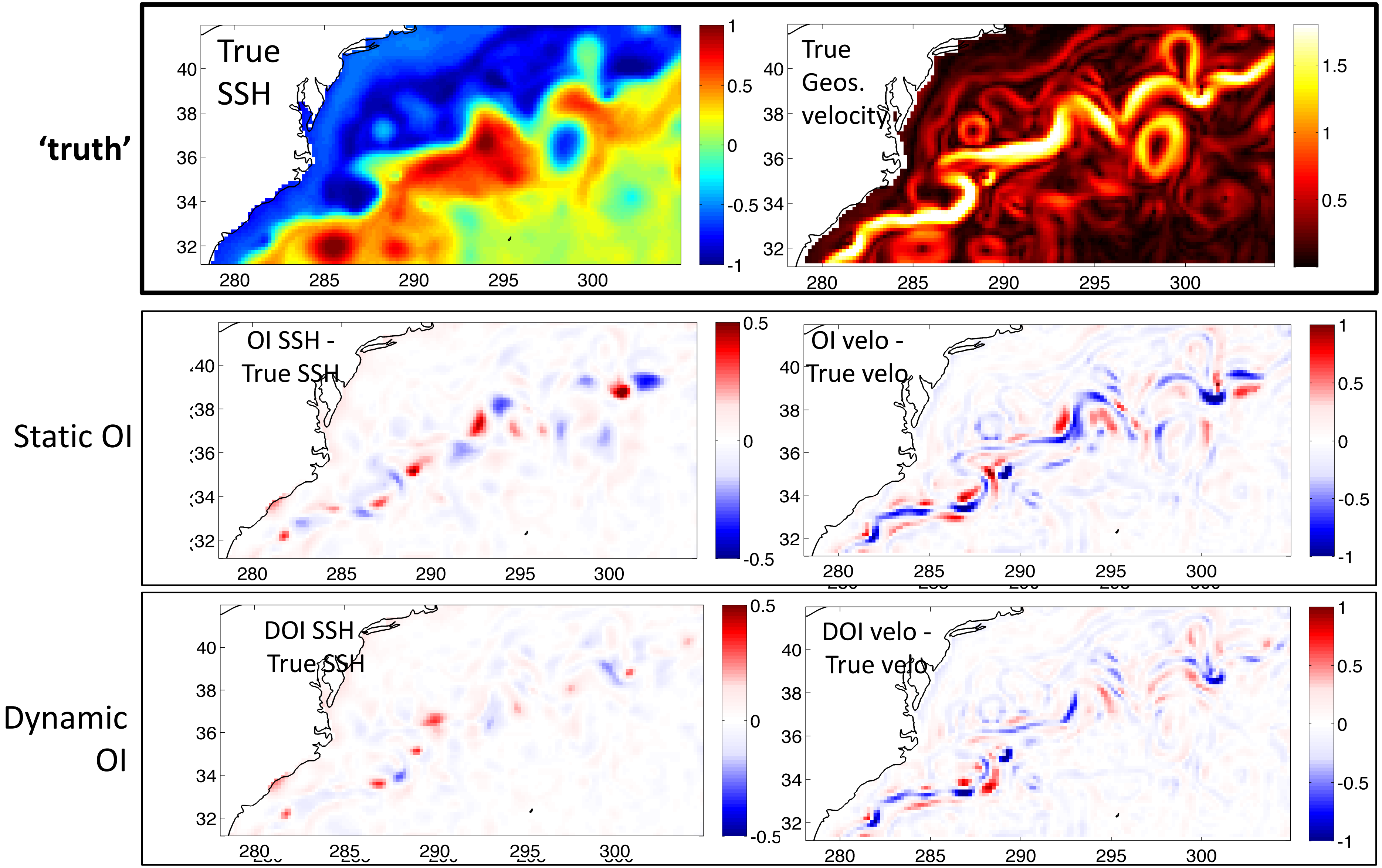
- Regional study in the Gulf Stream
- Reference SSH field: 'unknown truth' state from a MITgcm global simulation at ~6km resolution.

Constellation of 3 satellites on Jason (2) and AltiKa (1) orbits

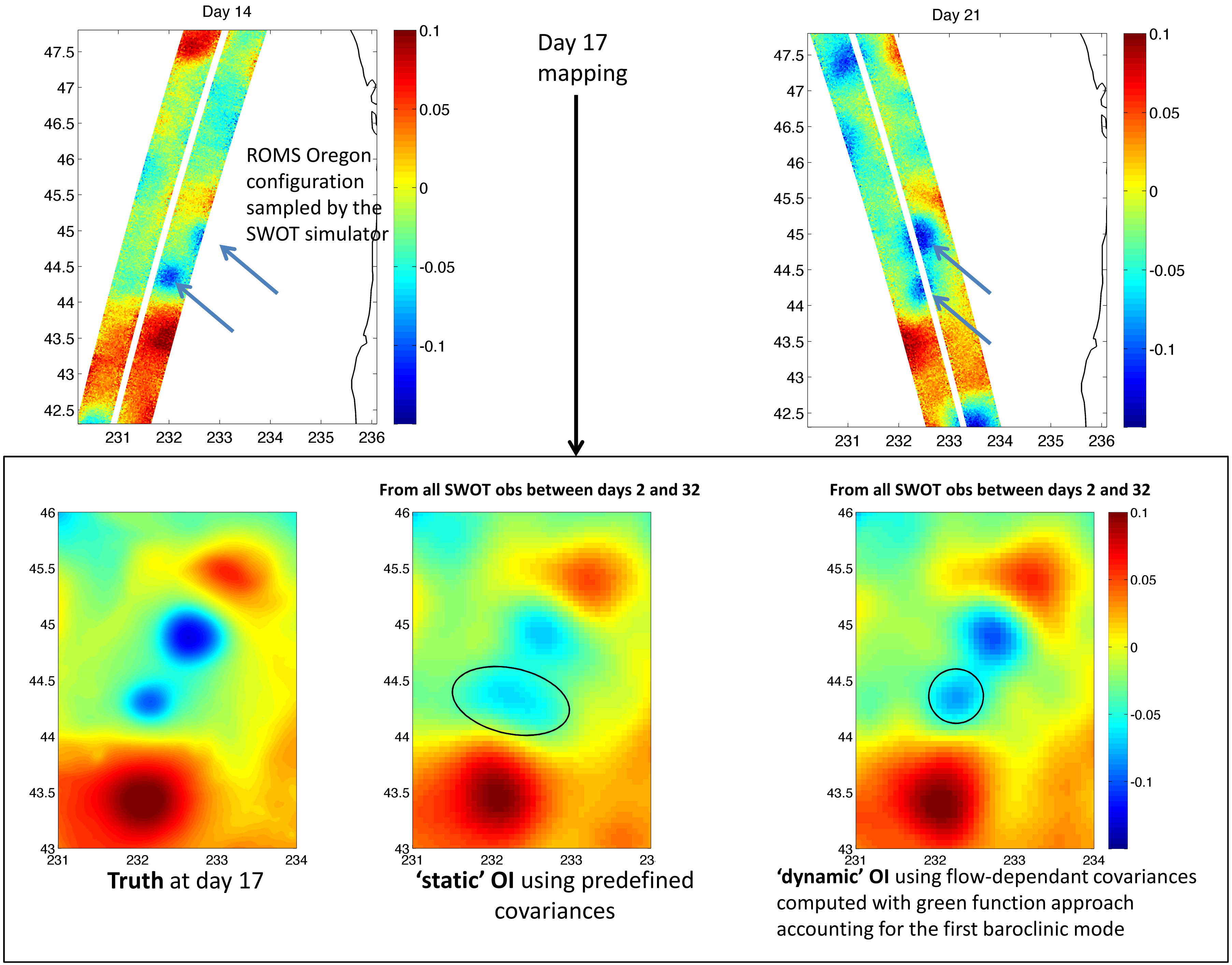


Along-track synthetic observations

Results



Error variance reduction by ~30-35% over 6 month worth of analysis



Day 14
ROMS Oregon configuration sampled by the SWOT simulator
Day 17 mapping
Day 21
0.1
0.05
0
-0.05
-0.1
From all SWOT obs between days 2 and 32
From all SWOT obs between days 2 and 32
Truth at day 17
'static' OI using predefined covariances
'dynamic' OI using flow-dependant covariances computed with green function approach accounting for the first baroclinic mode

Conclusions

- A first attempt, will be demonstrated with real obseravtions soon.

- Possible improvements of the propagator:

→Add impact of Ekman current?

→test SQG framework

→ Add cyclo-geostrophy in advection term?

- Altimetry-only so far, but does not exclude multi-sensor approaches for better constraints

- Others possible approaches to “fill” temporal gaps, e.g. optimal transport, …